Lesson 25 Laws of Exponents

I. Zero and Negative Exponents

How do Exponents Work?

1. $3 \bullet 3 = 3^2$ (2)
2. $\underbrace{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4}_{5} = 4^5$
3. $\underbrace{x \bullet x \bullet x}_{3} = x^3$

Base → What is being multiplied

Exponent → How many bases you are multiplying

Special Exponents

Zero as an Exponent: Any number (that is not zero) raised to the zero power is equal to 1

A. $5^0 = 1$ B. $(-3)^0 = 1$ C. $x^0 = 1$

Negative Exponents: Moving a base and its exponent across the fraction bar changes the sign of the exponent.

D. $2^{-1} = \frac{2^{-1}}{1} = \frac{1}{2^1}$ E. $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$ F. $\frac{1}{2^{-3}} = \frac{2^3}{1} = 8$

Examples: Simplify the expression so that there are no negative exponents left.

1. $(-1.23)^0 = 1$
2. $\frac{(-4)^{-3}}{1} = \frac{1}{(-4)^3}$
3. $\frac{2^3}{3^{-5}} = 2^3 \cdot 3^5$

Examples with Variables: Simplify the expression so that there are no negative exponents left.

4. $\frac{7s^{-4}t^2}{1}$ $= \frac{7t^2}{s^4}$
5. $\frac{2}{a^{-3}}$ $= 2a^3$
6. $\frac{n^{-5}}{v^2}$ $= \frac{1}{v^2 n^5}$

II. Multiplication Properties of Exponents

Rule: Multiplication Powers with the Same Base

For every nonzero number a and integers m and n, $a^m \cdot a^n = a^{m+n}$

Examples: $3^5 \cdot 3^4 = 3^{5+4} = 3^9$ $\quad \underbrace{3\cdot3\cdot3\cdot3\cdot3}_{5}\cdot\underbrace{3\cdot3\cdot3\cdot3}_{4} = 3^9$

$h^2 \cdot h^9 = h^{2+9} = h^{11}$ $\quad \underset{2}{h\cdot h}\cdot\underbrace{h\cdot h\cdot h\cdot h\cdot h\cdot h\cdot h\cdot h\cdot h}_{9} = h^{11}$

Examples: Simplify each expression.

7. $11^4 \cdot 11^3$ $= 11^{4+3} = 11^7$

8. $5^{-2} \cdot 5^2$ $= 5^{-2+2} = 5^0 = 1$

9. $7^{-3} \cdot 7^2 \cdot 7^6$ $= 7^{-3+2+6} = 7^5$

Examples with Variables: Simplify each expression.

10. $2n^5 \cdot 3n^{-2}$ $2 \cdot 3\, n^{5-2} = 6n^3$

11. $5x \cdot 2y^4 \cdot 3x^8$ $5 \cdot 2 \cdot 3\, x^{1+8} y^4 = 30 x^9 y^4$

12. $m^2 \cdot n^{-2} \cdot 7m^1$ $7m^{2+1} n^{-2} = \frac{7m^3 n^{-2}}{1} = \frac{7m^3}{n^2}$

III. More Multiplication Properties of Exponents

Rule: Raising a Power to a Power

For every nonzero number a and integers m and n, $(a^m)^n = a^{m \cdot n}$

Examples: Simplify each expression.

13. $(x^2)^5 = x^{10}$

14. $(a^{-4})^7$ $\frac{a^{-28}}{1} = \frac{1}{a^{28}}$

15. $c^5 \cdot (c^3)^{-2}$ $= c^5 \cdot c^{-6} = c^{5-6} = \frac{c^{-1}}{1} = \frac{1}{c}$

Rule: Raising a Product to a Power

For every nonzero number a and b and integer n, $(ab)^n = a^n b^n$

Examples: Simplify each expression.

16. $(3x)^4 = 3^4 x^4 = 81x^4$

17. $(5y)^3 = 5^3 y^3 = 125y^3$

Complex Examples: Simplify each expression.

18. $(x^{-2})^2 (3xy^2)^4$ $x^{-4}\, 3^4 x^4 y^8$ $81 x^{-4+4} y^8$ $= 81(x^0) y^8 = 81(1) y^8 = 81y^8$

19. $(2a^3)^5 (3ab^2)^3$ $= 2^5 a^{15} \cdot 3^3 a^3 b^6$ $= 32 \cdot 27\, a^{15+3} b^6$ $= 864\, a^{18} b^6$

IV. Division Properties of Exponents

Rule: Dividing Powers with the Same Base

For every nonzero number a and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$

Simply Rules to Follow:

1. Give each base its own fraction.
2. Always move the base with the smaller exponent

Examples: Simplify each expression.

20. $\frac{a^6}{a^{14}}$

$\frac{1}{a^{14-6}} = \frac{1}{a^8}$

21. $\left(\frac{c^{-1}d^3}{c^5d^{-4}}\right)$

$\frac{d^{3+4}}{c^{5+1}}$

$= \frac{d^7}{c^6}$

22. $\left(\frac{a^2b}{a^4b^3}\right)$

$= \frac{1}{a^{4-2}b^{3-1}}$

$= \frac{1}{a^2b^2}$

23. $\frac{3m^{-1}n^2}{5m^3n}$

$\frac{3}{5} \cdot \frac{m^{-1}}{m^3} \cdot \frac{n^2}{n^1}$

$\frac{3}{5} \cdot \frac{1}{m^{3+1}} \cdot \frac{n^{2-1}}{1}$

$\frac{3n}{5m^4}$