Lesson 25   Laws of Exponents

I. Zero and Negative Exponents

How do Exponents Work?

1. $3 \bullet 3 = 3^2$

   Base → What is being multiplied

2. $\underbrace{4 \bullet 4 \bullet 4 \bullet 4 \bullet 4}_{5} = 4^5$

   Exponent → How many bases you are multiplying

3. $\underbrace{x \bullet x \bullet x}_{3} = x^3$

Special Exponents

Zero as an Exponent: Any number (that is not zero) raised to the zero power is equal to 1

A. $5^0 = 1$

B. $(-3)^0 = 1$

C. $x^0 = 1$

Negative Exponents: Moving a base and its exponent across the fraction bar changes the sign of the exponent.

D. $2^{-1} = \dfrac{2^{-1}}{1} = \dfrac{1}{2^1}$

E. $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

F. $\dfrac{1}{2^{-3}} = \dfrac{2^3}{1} = 8$

Examples: Simplify the expression so that there are no negative exponents left.

1. $(-1.23)^0$

2. $(-4)^{-3}$

3. $\dfrac{2^3}{3^{-5}}$

Examples with Variables: Simplify the expression so that there are no negative exponents left.

4. $7s^{-4}t^2$

5. $\dfrac{2}{a^{-3}}$

6. $\dfrac{n^{-5}}{v^2}$

II.   Multiplication Properties of Exponents

Rule: Multiplication Powers with the Same Base

For every nonzero number a and integers m and n, $a^m \cdot a^n = a^{m+n}$

Examples: $3^5 \cdot 3^4 = 3^{5+4} = 3^9$ $\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{5} \cdot \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4} = 3^9$

                       $h^2 \cdot h^9 = h^{2+9} = h^{11}$ $\underbrace{h \cdot h}_{2} \cdot \underbrace{h \cdot h \cdot h \cdot h \cdot h \cdot h \cdot h \cdot h \cdot h}_{9} = h^{11}$

Examples: Simplify each expression.

7. $11^4 \cdot 11^3$ 8. $5^{-2} \cdot 5^2$ 9. $7^{-3} \cdot 7^2 \cdot 7^6$

Examples with Variables: Simplify each expression.

10. $2n^5 \cdot 3n^{-2}$ 11. $5x \cdot 2y^4 \cdot 3x^8$ 12. $m^2 \cdot n^{-2} \cdot 7m$

III.   More Multiplication Properties of Exponents

Rule: Raising a Power to a Power

For every nonzero number a and integers m and n, $(a^m)^n = a^{m \cdot n}$

Examples: Simplify each expression.

13. $(x^2)^5$ 14. $(a^{-4})^7$ 15. $c^5 \cdot (c^3)^{-2}$

Rule: Raising a Product to a Power

For every nonzero number a and b and integer n, $(ab)^n = a^n b^n$

Examples: Simplify each expression.

16. $(3x)^4$ 17. $(5y)^3$

Complex Examples: Simplify each expression.

18. $(x^{-2})^2 (3xy^2)^4$ 19. $(2a^3)^5 (3ab^2)^3$

IV. Division Properties of Exponents

Rule: Dividing Powers with the Same Base

For every nonzero number a and integers m and n, $\dfrac{a^m}{a^n} = a^{m-n}$

Simply Rules to Follow:
1. Give each base its own fraction.
2. Always move the base with the smaller exponent

Examples: Simplify each expression.

20. $\dfrac{a^6}{a^{14}}$

21. $\dfrac{c^{-1}d^3}{c^5 d^{-4}}$

22. $\dfrac{a^2 b}{a^4 b^3}$

23. $\dfrac{3m^{-1}n^2}{5m^3 n}$